Approximation Method of Analytic Hierarchy Process (AHP) as developed by Dr T.L. Saaty and others

Compiled and Described
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Step-I: Decomposition of problem into hierarchical structure

Decomposition of problem into hierarchic fashion into goal (which addresses the problem), criteria (on which subsequent level depend), and the list of alternatives at third level. Therefore three levels are minimum level; sub-criteria level may be introduced.

Step-II: Pair wise comparison

Judgment values to prepare judgment matrices

The pair wise comparison is made of criteria with respect to goal (hierarchical level-1), and of each alternative with respect to each criterion (hierarchical level-2), while filling matrix; the suitable indicators are identified for each element. Keeping these indicators into consideration and using a fundamental scale (Box-1), suitable values are allotted to various judgments giving rise to square matrix as in Box-2.

Box-1: Fundamental scale

Numerical scale	Verbal terms
1	Equally important
3	Moderately more important
5	Strongly more important
7	Very strongly more important
9	Extremely more important
2,4,6,8	Intermediate values to reflect compromise
Reciprocal scale i.e. 1/3, 1/5, 1/8 etc	Reciprocates the compared element in pair

Box-2

a_{11}	a_{12}	a_{13}	…a_{1n}
a_{21}	a_{22}	a_{23}	…a_{2n}
.	.	.	.
.	.	.	.
.	.	.	.
a_{n1}	a_{n2}	a_{n3}	a_{nn}

Observe reciprocal properties,
$a_{ji} = 1/ a_{ij}$, where,
i refer to row
j refer to column

Unlike physical objects (which can be judged through measurement means), the fundamental scale helps to judge intangible feelings and un-quantifiable factors by eliciting n (n-1)/2 judgments (Saaty and Kearns, 1985), and {n (n-1) j} pair wise comparisons, where n= total no. of elements being compared and j= total no. of criteria (Hartwich and Janssen, 2000). The relative element compared with itself is 1; therefore diagonal of matrix (upper left to lower right cells) contains 1's. The group judgments are given by consensus, in case of moderate confusions, use intermediate values, and in case of strong disagreements take geometric mean of individual judgments of group members (Forman and Peniwati, 1996).

Assigning relative weight to the elements

Geometric Mean (GM) of each row is called estimates of eigenvector component (Saaty, 1980), it is normalized to the unity by dividing each entry (GM of each row) by the

sum of all entry (sum of GM of all rows). The normalized value thus obtained is relative weight or local priority or normalized priority vector of each element (denoted by x_1, x_2 …..x_n) with respect to hierarchical level in question.

Step-III: Check consistency of local priority

Measures of inconsistency are inbuilt component, which provide information on violation of numerical (cardinal, a_{ij}, $a_{jk} = a_{ik}$) and transitive (ordinal) consistency.

Calculate maximum eigen value

Multiply first element of each row by the first element of the column of x's (relative weight) i.e. x_1, the second element in the row by second element of column x's i.e. x_2 and so on. Sum of all these obtained values provide one number 'y' for that row (Box-3). The maximum eigen value is sum of y_1, y_2, ...y_n and denoted by λ max.

Box-3

$a_{11*} x_{1+} \; a_{12*} x_{2+} \ldots a_{1n*} x_n = y_1$
$a_{21*} x_{1+} \; a_{22*} x_{2+} \ldots a_{2n*} x_n = y_2$
. . . . .
. . . . .
. . . . .
$a_{n1*} x_{1+} \; a_{n2*} x_{2+} \ldots a_{nn*} x_n = y_n$

Box- 4

Matrix Size (MS)- Random Index (RI)

MS	**1**	**2**	**3**	**4**	**5**
RI	0	0	0.58	0.90	1.12
MS	**6**	**7**	**8**	**9**	**10**
RI	1.24	1.32	1.41	1.45	1.49

Calculate Consistency Index (CI) and Consistency Ratio (CR)

CI= λ max-n/ n-1, where n is total number of elements being compared, CR= CI/ RI, where RI is random consistency number of same matrix size. RI is consistency index of a randomly generated pair wise comparison matrix, value of RI varies with no. of elements to be compared as given in Box-4. The CR provides a measure of the probability that matrix was filled in purely at random; it is a comparison between current matrix and a purely random answering of questions. Acceptability of CR is ≤ 0.1 (Harker, 1989), in some cases it can be tolerated up to 0.2, but never more than that (Saaty and Kearns, 1985). If CR is not acceptable than revise the judgments through more careful analysis. The research conducted at NAARM on agricultural projects reveals that moderate variation in CR does not affect the order of priority (Soam, 2004), it is not very serious matter where priority need not to be precise to the minute numerical value.

Step-IV: Synthesis to calculate global [Final] priority

Multiply the local priority by the corresponding criteria weight and results are summed up to obtain global priority (Braunschweig, 2000)

$Pl \sum_{m=1}^{M} Pl_m V_m$ With $\sum_{l=1}^{L} Pl_m = 1$ and $\sum_{m=1}^{M} V_m = 1$, where,

$Pl =$ Final prioritization of project l
$Pl_m =$ Prioritization of project l with respect to criteria m
$V_m =$ Weight of criteria m
$l =$ (1------L)
$m =$ (1-----M)